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CYLINDRICAL SHELLS OF BIMODULUS COMPOSITE MATERIAL

by

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INTRODUCTION

The expanding use of composite materials in structures has resulted in the requirement for more realistic mathematical models of their mechanical behavior and incorporation of these models into structural analyses. It has been found experimentally that certain fiber-reinforced materials, especially those with very soft matrices (for example, cord-rubber composites), have quite different elastic behavior depending upon whether the fiber-direction strain is tensile or compressive (7,8,10,23). As a first approximation, the stress-strain behavior of such materials is often modeled as being bilinear, with different slopes (elastic properties) depending upon the sign of the fiber-direction strain. This kind of material is termed a <u>bimodulus</u> composite material, and it has been demonstrated that the fiber-governed symmetric-compliance model proposed by the senior author (7) agrees well with experimental data obtained from uniaxial tests on several materials with severe bimodulus action.

The literature available in English on shell-type analyses of structures constructed of bimodulus materials is limited to various special cases. Ambartsumyan treated a variety of <u>isotropic</u>-bimodulus-material shells: axisymmetrically loaded circular cylindrical shells (1), membrane shells with cylindrical, conical, and spherical meridians (2), torsion of circular-cylindrical shells (3), axisymmetrically loaded weak-moment[3] shells of revolution (4,5). Jones (13) considered buckling of isotropic circular cylindrical shells, and Jones (14) and Jones and Morgan (15) analyzed buckling of orthotropic circular cylindrical shells.

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[1]Benjamin H. Perkinson Professor of Engineering, The Univ. of Oklahoma, Norman, OK.
[2]Graduate Research Asst., Mechanical Engineering, The Univ. of Oklahoma, Norman, OK.
[3]Ambartsumyan (4,5) defined a weak-moment shell as one in which the bending-stress components never exceed the corresponding membrane-stress components.

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Kamiya analyzed the one-dimensional problem of bending of an infinitely long cylindrical panel (16). More recently he presented iterative solutions for uniformly pressurized isotropic (17) and orthotropic (18) circular cylindrical shells.

In the present work a closed-form solution is presented for a circular cylindrical shell cross-plied of orthotropic bimodulus material. The shell may be either a cylindrically curved panel freely supported on all four edges or a circumferentially complete cylinder freely supported at each end.

In the realm of cylindrical shells, there is a great variety of theories; see (20). However, it is now generally agreed that the Sanders "best" first-approximation theory is the most accurate one for engineering purposes (12). Here, following the philosophy introduced in (6), we use four tracers to handle four popular theories as special cases of the same general theory: Sanders' best (24), Love's first approximation (21), Morley (22), and Donnell (11).

FORMULATION

Laminate Behavior (19).—First, let us consider the case of a flat plate of ordinary (not bimodulus) material. A single-layer plate made of a macroscopically homogeneous ordinary material is obviously symmetric about the plate midplane; thus, when it undergoes in-plane stretching or small-deflection bending, there is no coupling between bending and stretching. A plate comprised of multiple layers of ordinary materials also has no bending-stretching coupling provided that the layers are arranged symmetrically about the midplane of the laminate. Conversely, a general laminate without midplane symmetry exhibits bending-stretching coupling.

Next, let us consider a single layer of bimodulus material. The different elastic moduli in tension and compression cause a shift in the neutral surface away from the geometric midplane, and thus midplane symmetry cannot exist. In other words, a single-layer bimodulus-material plate exhibits bending-stretching coupling of the orthotropic type, analogous to a two-layer cross-ply plate

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(which has one layer at 0° and the other at 90°) of ordinary orthotropic material. Further, a multiple-layer cross-ply plate consisting of bimodulus orthotropic materials also exhibits orthotropic bending-stretching coupling.

Of course, when one considers a curved shell (as in the present work), there is an additional bending-stretching action (even for homogeneous, isotropic ordinary materials) due to the effect of the shell curvature.

Applying the fiber-governed symmetric-matrix macroscopic material model (7), one assumes that there are _two_ symmetric plane-stress reduced stiffness matrices: one governing when the fiber-direction normal strain is tensile and the other when this strain is compressive. Thus, the generalized Hooke's law may be written as

$$
\left\{ \begin{array}{c} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{array} \right\} = \left[\begin{array}{ccc} Q_{11k\ell} & Q_{12k\ell} & 0 \\ Q_{12k\ell} & Q_{22k\ell} & 0 \\ 0 & 0 & Q_{66k\ell} \end{array} \right] \left\{ \begin{array}{c} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{array} \right\} \tag{1}
$$

The nomenclature is listed in Appendix II. Following (19), the contracted *notation of classical composite-material mechanics* is used here, with the addition of subscript k (=1 or 2 for fiber-direction strain that is tensile or compressive, respectively) and ℓ, which denotes the layer number (=1 to N, the total number of layers). From Eq. 1, one can see that there are now eight independent elastic stiffness coefficients for each layer (a set of four for tension and another set of four for compréssion). This is in distinct contrast to a thin layer of ordinary orthotropic material which has a total of only four independent coefficients.

The stretching, bending-stretching coupling, and bending stiffnesses of the laminate are formally defined in exactly the same manner as in the case of a laminate of ordinary material:

$$
A_{ij} \equiv \int_{-h/2}^{h/2} Q_{ijk\ell} \, dz \quad ; \quad B_{ij} \equiv \int_{-h/2}^{h/2} z Q_{ijk\ell} \, dz
$$

$$
D_{ij} \equiv \int_{-h/2}^{h/2} z^2 Q_{ijk\ell} \, dz \tag{2}
$$

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Now, in addition to carrying out the integrations in a piecewise fashion from layer to layer, one also has to take into consideration the possibility of different properties (tension or compression) within a layer. The results for a two-layer cross-ply bimodulus laminate are presented in Appendix III. The constitutive relations are

$$
\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11}` & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \\ \kappa_1 \\ \kappa_2 \\ \kappa_6 \end{Bmatrix}
$$

(3)

where ε_j^0 and κ_j are related to the total strains ε_j as follows

$$\varepsilon_j = \varepsilon_j^0 + z\kappa_j$$

(4)

Thin-Shell Behavior.— Letting x and y denote the axial and circumferential position coordinates on the middle surface (see Fig. 1), one can write the equilibrium equations in the absence of body forces and body moments as

$$N_{1,x} + N_{6,y} - (C_4/R)\, M_{6,y} = 0$$
$$N_{6,x} + N_{2,y} + (C_3/R)\, M_{6,x} + (C_1/R)\, M_{2,y} = 0$$
$$M_{1,xx} + 2M_{6,xy} + M_{2,yy} - (N_2/R) + p = 0$$

(5)

Here $(\ )_{,xy}$ denotes $\partial^2(\ )/\partial x \partial y$, and C_ℓ are tracers (see Eqs. 6 and Table 1).

TABLE 1.   List of Shell-Theory Tracers

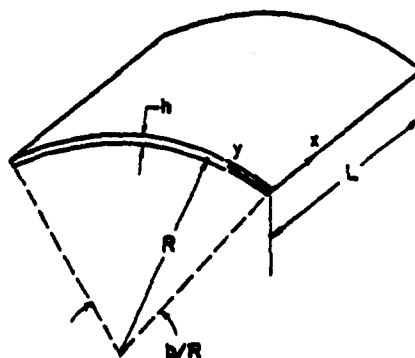| Theory (in descending order)	C_1	C_2	C_3	C_4
Sanders	1	1	3/2	1/2
Love's 1st Approximation	1	1	1	0
Morley	1	0	0	0
Donnell	0	0	0	0

Fig. 1   Shell geometry.

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For a thin shell (Kirchhoff-Love hypothesis) undergoing small deflections,

$$\varepsilon_1^0 = u,_x \quad ; \quad \varepsilon_2^0 = v,_y + (w/R) \quad ; \quad \varepsilon_6^0 = u,_y + v,_x$$

$$\kappa_1 = - w,_{xx} \quad ; \quad \kappa_2 = - w,_{yy} + (C_2/R) v,_y \tag{6}$$

$$\kappa_6 = - 2w,_{xy} + (C_3/R) v,_x - (C_4/R) u,_y$$

Substituting Eqs. 3 and 6 into Eqs. 5, one obtains the following set of governing partial differential equations in the midplane displacements u,v,w:

$$[L_{rs}]\{u,v,w\}^T = \{0,0,p\}^T \qquad (r,s=1,2,3) \tag{7}$$

The elements of the linear differential operator matrix are:

$$L_{11} = A_{11}d_x^2 + (A_{66} - 2C_4B_{66}R^{-1} + C_4^2D_{66}R^{-2})d_y^2$$

$$L_{12} = [A_{12} + A_{66} + C_2B_{12}R^{-1} + (C_3 - C_4)B_{66}R^{-1} - C_3C_4D_{66}R^{-2}]d_xd_y$$

$$L_{13} = - B_{11}d_x^3 - (B_{12} + 2B_{66} - 2C_4D_{66}R^{-1})d_xd_y^2 + \underline{A_{12}R^{-1}d_x} = L_{31}$$

$$L_{21} = [A_{12} + A_{66} + C_1B_{12}R^{-1} + (C_3 - C_4)B_{66}R^{-1} - C_3C_4D_{66}R^{-2}]d_xd_y$$

$$L_{22} = (A_{66} + 2C_3B_{66}R^{-1} + C_3^2D_{66}R^{-2})d_x^2$$

$$+ [A_{22} + (C_1 + C_2)B_{22}R^{-1} + C_1C_2D_{22}R^{-2}]d_y^2 \tag{8}$$

$$L_{23} = - [B_{12} + 2B_{66} + C_1D_{12}R^{-1} + 2C_3D_{66}R^{-1}]d_x^2d_y$$

$$- (B_{22} + C_1D_{22}R^{-1})d_y^3 + \underline{(A_{22} + C_1B_{22}R^{-1})R^{-1}d_y}$$

$$L_{32} = - [B_{12} + 2B_{66} + C_2D_{12}R^{-1} + 2C_3D_{66}R^{-1}]d_x^2d_y$$

$$- (B_{22} + C_2D_{22}R^{-1})d_y^3 + \underline{(A_{22} + C_2B_{22}R^{-1})R^{-1}d_y}$$

$$L_{33} = D_{11}d_x^4 + 2(D_{12} + 2D_{66})d_x^2d_y^2 + D_{22}d_y^4 - \underline{2B_{12}R^{-1}d_x^2} - \underline{2B_{22}R^{-1}d_y^2} + \underline{A_{22}R^{-2}}$$

The only terms having a more complicated differential-equation structure, i.e., involving <u>additional</u> <u>operators</u> over those for a cross-ply bimodulus plate (19) are the six terms underlined in the above set. It is also noted that the L_{rs} are symmetric for each of the theories except Morley's.

<u>Criterion for Homogeneity Along the Middle Surface</u>.— In the derivation of Eqs. 7 and 8, it was tacitly assumed that the laminate stiffnesses (A_{ij}, B_{ij}, D_{ij}) are all independent of the location (x,y) along the middle surface. Thus, in view of the role of the fiber-direction neutral-surface position (z_{nx} for a single layer with fibers oriented axially; z_{nx} and z_{ny} for a cross-ply laminate) in determining these stiffnesses, it is necessary that the governing values of z_n (z_{nx} or z_{ny} or both) be independent of x and y.

Using Eqs. 4 and 6, one finds that these requirements are, for fibers oriented axially:

$$z_{nx} = -\epsilon_1^0/\kappa_1 = u,_x/w,_{xx} = \text{const.} \tag{9}$$

For fibers oriented circumferentially:

$$z_{ny} = -\epsilon_2^0/\kappa_2 = (v,_y + R^{-1}w)/(w,_{yy} - C_2 R^{-1} v,_y) = \text{const.} \tag{10}$$

<u>Boundary Conditions and Loading</u>.— Freely supported edges at x=0 and x=L (where L is the shell length) require:

$$N_1(0,y) = N_1(L,y) = 0 \quad ; \quad v(0,y) = v(L,y) = 0$$
$$w(0,y) = w(L,y) = 0 \quad ; \quad M_1(0,y) = M_1(L,y) = 0 \tag{11}$$

If the shell is circumferentially complete, there is no longitudinal "edge", so that the only additional requirements on the displacements u,v,w are that they each be periodic with respect to y.

However, if the shell is a panel (not circumferentially complete) of circumferential arc length b, the following additional freely supported edge boundary conditions must be met:

$$N_2(x,0) = N_2(x,b) = 0 \quad ; \quad u(x,0) = u(x,b) = 0$$

$$w(x,0) = w(x,b) = 0 \quad ; \quad M_2(x,0) = M_2(x,b) = 0 \tag{12}$$

The loading is assumed to be sinusoidally distributed normal pressure:

$$p = p_0 \sin \alpha x \sin \beta y \tag{13}$$

where $\alpha = \pi/L$, and $\beta = 1/R$ for a circumferentially complete cylinder and $\beta = \pi/b$ for a panel.

SOLUTION

<u>Exact Closed-Form Solution</u> .—Following the Love's first-approximation theory analysis of Timoshenko and Woinowsky-Krieger (25) for a homogeneous, isotropic ordinary-material shell of the same geometry, loading, and boundary conditions, we take the solution to be of the form

$$u(x,y) = U \cos \alpha x \sin \beta y \quad ; \quad v(x,y) = V \sin \alpha x \cos \beta y$$

$$w(x,y) = W \sin \alpha x \sin \beta y \tag{14}$$

Substituting Eqs. 13 and 14 into matrix differential Eq. 7, one obtains the following linear matrix algebraic equation in the displacement coefficients U, V, W:

$$[C_{rs}]\{U,V,W\}^T = \{0,0,p_0\}^T \tag{15}$$

where

$$C_{11} = A_{11}\alpha^2 + (A_{66} - 2\bar{C}_4 B_{66} + \bar{C}_4^2 D_{66})\beta^2$$

$$C_{12} = [A_{12} + A_{66} + \bar{C}_2 B_{12} + (\bar{C}_3 - \bar{C}_4)B_{66} - \bar{C}_3\bar{C}_4 D_{66}]\alpha\beta$$

$$C_{13} = - B_{11}\alpha^3 - (B_{12} + 2B_{66} - 2\bar{C}_4 D_{66})\alpha\beta^2 - A_{12}R^{-1}\alpha$$

$$C_{21} = [A_{12} + A_{66} + \bar{C}_1 B_{12} + (\bar{C}_3 - \bar{C}_4)B_{66} - \bar{C}_3\bar{C}_4 D_{66}]\alpha\beta \tag{16}$$

$$C_{22} = (A_{66} + 2\bar{C}_3 B_{66} + \bar{C}_3^2 D_{66})\alpha^2 + [A_{22} + (\bar{C}_1 + \bar{C}_2)B_{22} + \bar{C}_1\bar{C}_2 D_{22}]\beta^2$$

$$C_{23} = - (B_{12} + 2B_{66} + \bar{C}_1 D_{12} + 2\bar{C}_3 D_{66})\alpha^2\beta - (B_{22} + \bar{C}_1 D_{22})\beta^3 - (A_{22} + \bar{C}_1 B_{22})R^{-1}\beta$$

$$C_{31} = C_{13}$$

$$C_{32} = - (B_{12} + 2B_{66} + \bar{C}_2 D_{12} + 2\bar{C}_3 D_{66})\alpha^2\beta - (B_{22} + \bar{C}_2 D_{22})\beta^3 - (A_{22} + \bar{C}_2 B_{22})R^{-1}\beta$$

$$C_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 + A_{22}R^{-2} + 2B_{12}R^{-1}\alpha^2 + 2B_{22}R^{-1}\beta^2$$

$$\bar{C}_i = C_i R^{-1}$$

Substituting solution Eqs. 14 into Eqs. 9 and 10, one obtains

$$z_{nx} = U/W\alpha = \text{const.} \tag{17}$$

$$z_{ny} = -\frac{1 - (\beta RV/W)}{\beta[\beta R - (C_2V/W)]} = \text{const.} \tag{18}$$

Since differential Eqs. 7, homogeneity conditions 9 and 10, and boundary conditions 11 and 12 are all satisfied by solution Eqs. 14, they constitute an exact closed-form solution. However, the resulting algebraic structure is so lengthy that is was found to be most expedient to solve the system of algebraic equations, Eqs. 15-18, by iterating z_{nx} and z_{ny}.

Numerical Results.—Before any numerical results are presented, it is convenient to define the following dimensionless quantities:

$$U^* = UE_{22_c}h^3/p_0b^4, \quad V^* = VE_{22_c}h^3/p_0b^4 ,$$

$$W^* = WE_{22_c}h^3/p_0b^4, \quad Z_x = z_{nx}/h \quad , \quad Z_y = z_{ny}/h \tag{19}$$

Since the present investigators are unaware of any numerical results for cylindrical shells or panels of the type considered here, the best comparison with previous work is to investigate the convergence of the present results, as the radius-to-thickness ratio, R/h, is increased, to those for rectangular plates of the same material (9). This is done in Fig. 2 for both axially oriented (0°) single-layer and two-layer cross-ply (0°/90°) aramid-rubber, using elastic properties listed in Table 2. The Ref. (9) plate-theory results were obtained from a closed-form solution and were shown in (9) to agree very well with results obtained by the energy method and by the mixed finite-element method.

Inspection of Fig. 2 shows that indeed the quantities presented do converge to the corresponding flat-plate values. It is interesting to note that both Z_x and W^* increase as R/h is increased, while $-Z_y$ decreases. Furthermore, it is observed that there is very little difference between the single-layer and cross-ply values of Z_x and W^*, but there is considerable difference
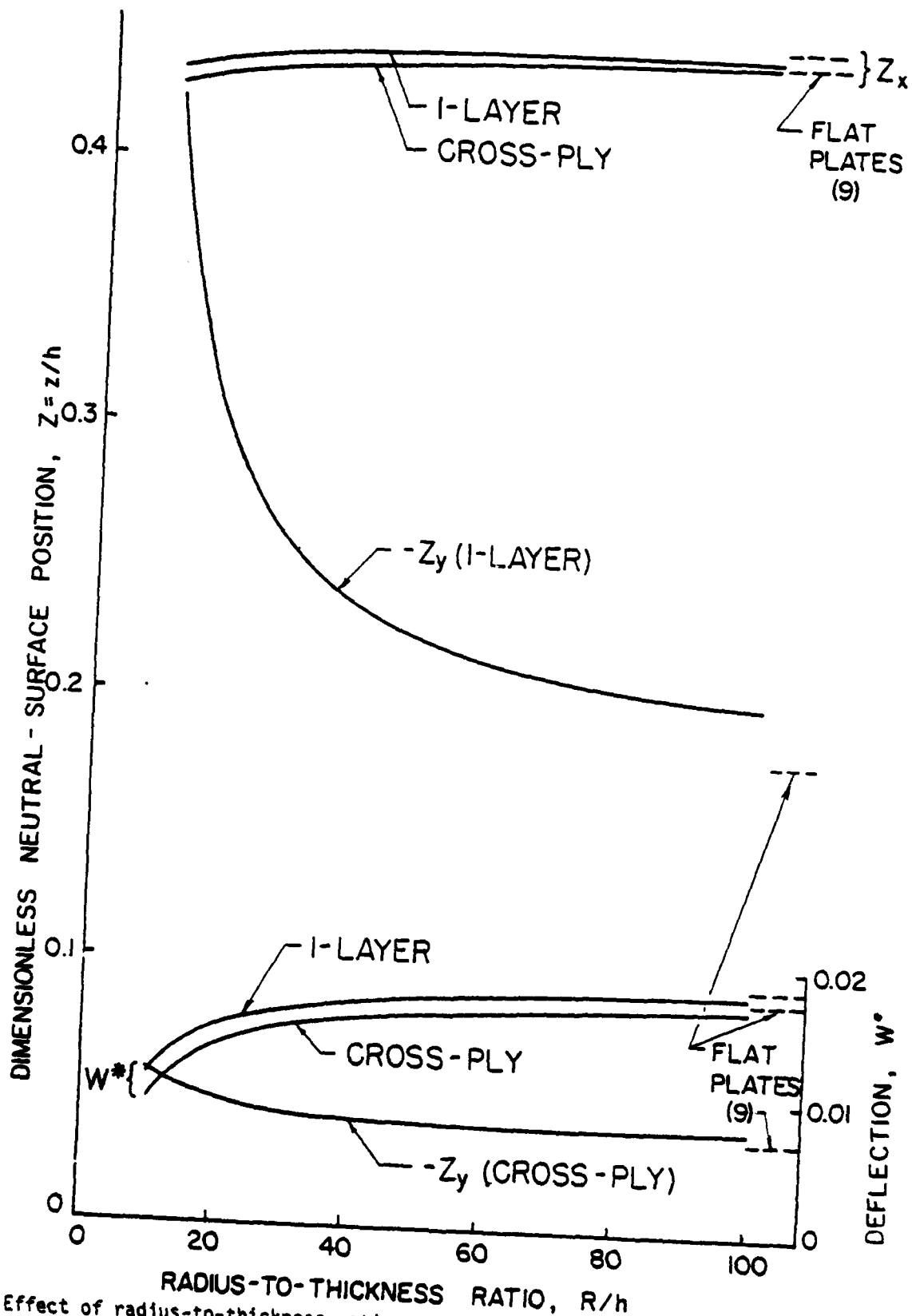
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Fig. 2 Effect of radius-to-thickness ratio on neutral-surface position and dimensionless center deflection of square (a/b=1) panels of single-layer (0°) and cross-ply (0°/90°) aramid-rubber by Sanders' theory.

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TABLE 2.—Elastic Properties of Two Fiber-Reinforced Bimodulus
Composite Materials[1]

Property	Aramid-Rubber	Polyester-Rubber
Tension Properties:		
Major Young's Modulus, psi (GPa)	519,000 (3.58)	89,500 (0.617)
Minor Young's Modulus, psi (MPa)	1,320 (9.09)	1,160 (8.00)
Major Poisson's Ratio, dimensionless	0.416	0.475
Shear Modulus, psi (MPa)	537 (3.70)	380 (2.62)
Compression Properties:		
Major Young's Modulus, psi (MPa)	1,740 (12.0)	5,350 (36.9)
Minor Young's Modulus, psi (MPa)	1,740 (12.0)	1,540 (10.6)
Major Poisson's Ratio, dimensionless	0.205	0.185
Shear Modulus, psi (MPa)	537 (3.70)	387 (2.67)

[1] Based on experimental data reported in (23), based on mathematical model in (7).

between these values of Z_y.

The effect of panel aspect ratio, L/b, on these same quantities for aramid-rubber panels is shown in Fig. 3. Again the very small differences between single-layer and cross-ply results are noted for Z_x and W^* but not for $-Z_y$.

The results presented in Figs. 2 and 3 were based on Sanders' shell theory. However, it was found that there was a very negligible difference (<< 1%) among the results based on Sanders', Love's first approximation, and Morley's shell theories. For certain conditions there were considerable differences between the Sanders and Donnell theory results. These are tabulated in Tables 3 and 4 for single-layer and cross-ply panels of various R/h and L/b ratios and constructed of either aramid-rubber or polyester-rubber.

It can be seen in Tables 3 and 4, that with two exceptions (V^* for aramid-rubber at R/h=10, L/b=0.5 and for polyester-rubber at R/h=20, L/b=0.5), the Donnell error is larger in cross-ply panels. Also, it generally increases with increasing L/b and decreases with increasing R/h. For the same material,
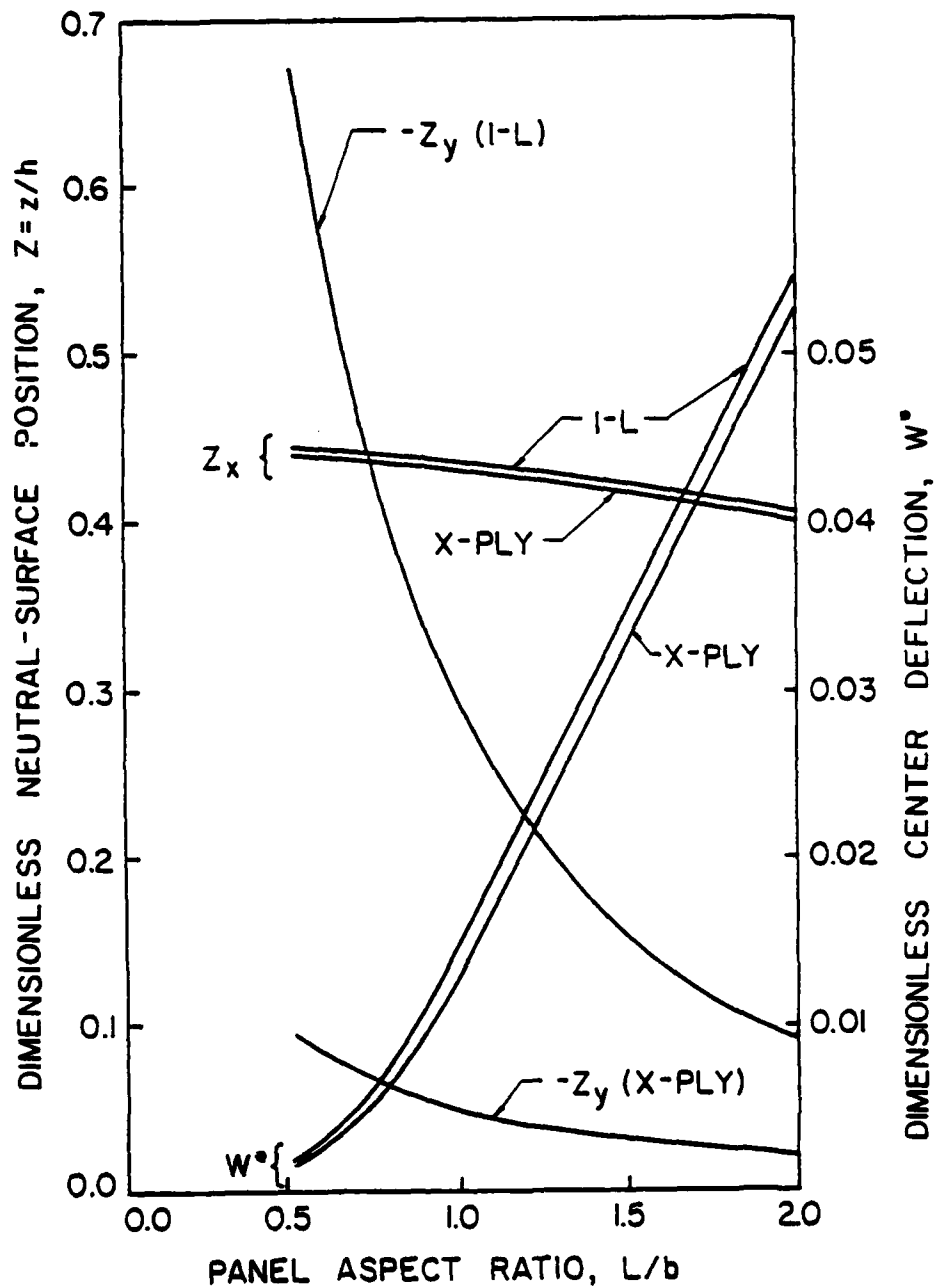
Fig. 3 Effect of panel aspect ratio on neutral-surface position
and dimensionless center deflection of 0° single-layer (1-L) and
0°/90° cross-ply (X-PLY) aramid-rubber panels with R/h=20 by
Sanders' theory.

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TABLE 3.—Dimensionless Displacements for Circular Cylindrical Panels
of Single-Layer, Axially Oriented, Orthotropic Bimodulus Material
by Sanders' Theory and Deviations[1] from it by Donnell's Theory

R/h	L/b	$U^* \times 10^4$	% Dev.	$V^* \times 10^4$	% Dev.	$W^* \times 10^4$	% Dev.
			Aramid-Rubber:				
10	0.5	4.703	0.04	0.3605	22.9	16.81	0.07
	1.0	14.96	1.80	21.36	3.62	110.0	1.86
	1.5	21.32	3.87	63.04	5.12	246.9	3.97
	2.0	24.68	5.94	113.2	7.05	403.33	6.08
20	0.5	5.090	-0.12	-0.9751	-4.74	18.18	-0.11
	1.0	20.55	0.43	10.39	3.02	149.5	0.47
	1.5	31.03	1.28	39.66	2.68	350.3	1.34
	2.0	34.99	2.03	71.96	3.19	548.4	2.11
50	0.5	5.228	-0.09	-1.843	-1.09	18.67	-0.08
	1.0	24.04	-0.03	-0.9313	-13.3	173.7	-0.02
	1.5	37.93	3.34	11.71	2.37	421.0	0.16
	2.0	42.15	0.29	26.69	1.70	642.5	0.33
			Polyester-Rubber:				
10	0.5	1.494	0.06	0.5934	6.38	7.828	0.10
	1.0	7.713	1.24	18.42	2.99	84.07	1.38
	1.5	14.11	3.41	66.70	4.89	249.7	3.71
	2.0	17.43	5.78	131.4	7.36	458.8	6.31
20	0.5	1.541	-0.03	-0.0420	-46.3	8.066	-0.005
	1.0	9.208	0.29	8.895	2.29	98.57	0.35
	1.5	18.76	1.02	38.53	2.42	315.3	1.15
	2.0	24.06	1.83	77.56	3.13	574.3	2.04
50	0.5	1.556	-0.03	-0.4386	-1.86	8.144	-0.02
	1.0	9.926	-0.00	1.354	5.38	105.1	0.03
	1.5	21.44	0.10	12.33	1.86	349.5	0.15
	2.0	28.18	0.23	28.13	1.59	634.8	0.31

[1] %Dev. = 100 x (Sanders-Donnell)/Sanders.

TABLE 4.—Dimensionless Displacements for Circular Cylindrical Panels
of Cross-Ply Orthotropic Bimodulus Material by Sanders' Theory
and Deviations[1] from it by Donnell's Theory

R/h	L/b	$U^* \times 10^4$	% Dev.	$V^* \times 10^4$	% Dev.	$W^* \times 10^4$	% Dev.
				Aramid-Rubber:			
10	0.5	3.694	0.72	3.800	1.34	13.35	0.73
	1.0	12.45	2.31	28.14	2.67	93.21	2.33
	1.5	19.32	4.31	69.95	4.59	227.3	4.34
	2.0	23.31	6.35	119.9	6.61	386.3	6.40
20	0.5	4.460	0.21	2.087	0.56	16.06	0.21
	1.0	18.10	0.85	19.33	1.07	133.4	0.86
	1.5	28.90	1.64	49.50	1.84	330.3	1.66
	2.0	33.60	2.35	81.08	2.55	532.3	2.37
50	0.5	4.849	0.002	0.6657	0.30	17.42	0
	1.0	22.06	0.17	8.240	0.32	161.0	0.16
	1.5	36.04	0.31	22.43	0.47	404.1	0.31
	2.0	40.93	0.43	36.27	0.61	629.5	0.44
				Polyester-Rubber:			
10	0.5	3.515	0.75	3.922	2.00	15.51	0.75
	1.0	11.99	3.15	31.72	4.27	112.5	3.29
	1.5	16.66	6.44	74.30	7.64	256.4	6.72
	2.0	17.07	9.16	114.9	10.5	392.2	9.64
20	0.5	4.041	0.17	1.818	0.98	17.72	0.17
	1.0	16.30	1.02	18.55	1.73	148.2	1.07
	1.5	22.72	2.07	42.71	2.80	326.4	2.16
	2.0	22.38	2.77	60.89	3.54	457.8	2.91
50	0.5	4.267	-0.005	0.263	1.44	18.65	0
	1.0	18.85	0.11	5.347	0.76	168.2	0.12
	1.5	26.46	0.26	13.17	0.85	364.8	0.28
	2.0	25.74	0.34	18.50	0.92	491.0	1.67

[1] %Dev. = 100 x (Sanders-Donnell)/Sanders.

lamination scheme, and geometry, the largest error is usually in V^*, and the least in U^*. For R/h=20, Table 5 shows that the dimensionless deflections W^* predicted using Love's first-approximation theory coincides with that using Sanders', and that the deviation of Morley-theory predictions is about one-half of that of Donnell's.

CONCLUDING REMARKS AND SUGGESTIONS FOR FURTHER RESEARCH

The exact closed-form solution presented here for small deflections of circular cylindrical shells of bimodulus composite materials has been shown to converge to an existing closed-form solution for rectangular plates as the radius-to-thickness ratio is increased. Thus, the present results are available as benchmarks for use in validating finite-element solutions.

The effects of laminate configuration, material, geometric parameters, and shell theory were all investigated. For the range of parameters investigated here, very close agreement was found among the results predicted on the basis of the Sanders', Love's first approximation, and Morley theories. However, the results obtained on the basis of Donnell's shallow-shell theory were not reliable except for very thin shells (R/h $\geq$ 20).

The following suggestions for further research on bimodulus composite-material shells constitute extensions of the research reported here:

1. Analysis of thermal-expansion effects due to a thermal gradient through the thickness as well as a mean temperature change

2. Free-vibration analysis

3. Buckling analysis

4. Extension to doubly curved shells

Current work is under way on all four of the above problems.

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TABLE 5.—Comparison of Dimensionless Displacements for Circular
Cylindrical Panels of Cross-Ply Orthotropic Material and
R/h=20 as Determined by Four Different Shell Theories

L/b	Sanders' & Love's 1st W^* x 10^4	Morley's W^* x 10^4	Morley's % Dev.[1]	Donnell's W^* x 10^4	Donnell's % Dev.[1]
		Aramid-Rubber:			
0.5	16.06	16.03	0.19	16.03	0.21
1.0	133.4	132.6	0.60	132.2	0.86
1.5	330.3	326.9	1.03	324.8	1.66
2.0	532.3	525.2	1.33	519.6	2.37
3.0	844.9	829.7	1.80	816.5	3.36
4.0	1039.3	1017.9	2.06	998.4	3.94
5.0	1159.7	1134.1	2.21	1110.3	4.26
6.0	1226.2	1198.0	2.30	1171.5	4.46
7.0	1281.6	1251.6	2.34	1222.9	4.58
8.0	1319.9	1288.4	2.39	1258.4	4.66
9.0	1347.6	1315.2	2.40	1284.1	4.71
10.0	1367.9	1334.8	2.42	1302.9	4.75
		Polyester-Rubber:			
0.5	17.72	17.69	0.17	17.69	0.17
1.0	148.2	147.3	0.61	146.6	1.07
1.5	326.4	322.5	1.20	319.3	2.16
2.0	457.8	450.7	1.55	444.5	2.91
3.0	595.2	583.9	1.90	573.6	3.63
4.0	655.1	641.9	2.01	629.4	3.92
5.0	681.5	667.3	2.08	653.8	4.06
6.0	700.3	685.5	2.11	671.4	4.13
7.0	711.6	696.5	2.12	682.0	4.16
8.0	718.9	703.6	2.13	688.8	4.19
9.0	723.8	708.4	2.13	693.4	4.20
10.0	727.2	711.6	2.14	696.5	4.22

[1]% Dev. = 100 x (Sanders-Other)/Sanders

ACKNOWLEDGMENTS

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

A_{ij} = stretching stiffness;

B_{ij} = bending-stretching coupling stiffness;

b = circumferential arc width of panel;

$C_\ell, \bar{C}_\ell$ = shell-theory tracer, C_ℓ/R;

C_{rs} = coefficients defined in Eqs. 16;

D_{ij} = bending stiffness;

E_{22_c} = Young's modulus transverse to the fibers, when the fiber-direction strain is < 0;

d_x = $\partial(\ )/\partial x$;

h = total shell thickness;

L = axial length of shell;

L_{rs} = linear differential operators defined in Eqs. 8;

M_i, N_i = stress couples and stress resultants;

p = radial pressure (positive outward);

p_o = magnitude of p at $x=L/2$, $y=b/2$;

$Q_{ijk\ell}$ = plane-stress-reduced stiffnesses of layer ℓ relating σ_i to ε_j for fiber-direction strain tensile (k=1) or compressive (k=2);

R = radius of shell middle surface;

U,V,W = displacement coefficients defined in Eqs. 14;

U^*,V^*,W^* = dimensionless displacement coefficients defined in Eqs. 19;

u,v,w = displacements in x,y,z directions;

x,y,z = position coordinates in axial, circumferential, and radial (measured positive outward from middle surface) directions;

Z_x, Z_y = z_{nx}/h, z_{ny}/h;

z_{nx}, z_{ny} = neutral-surface positions (measured from middle surface) for $\varepsilon_x=0$ and $\varepsilon_y=0$;

α, β = π/L, π/b for panel or $1/R$ for circumferentially complete cylindrical shell;

$\varepsilon_j, \varepsilon_j^0$ = strain component at arbitrary location (x,y,z) and at the middle surface $(x,y,0)$;

κ_j = curvature component;

σ_i = stress component.

APPENDIX III.—LAMINATE STIFFNESSES FOR TWO-LAYER CROSS-PLY LAMINATE OF BIMODULUS MATERIAL

For each specific laminate, it is necessary to evaluate the integral forms defining the laminate stiffnesses, Eqs. 2. This was accomplished in (19) for the case of a two-layer cross-ply laminate, i.e., one having one orthotropic layer (layer 1, located from $z=0$ to $z=h/2$) oriented in the x direction and the other (layer 2, located from $z=-h/2$ to $z=0$) oriented in the y direction.

The results obtained in (19) for $1/2 > Z_x > 0$ and $-1/2 < Z_y < 0$ are:

$$A_{ij}/h = (1/2)(Q_{ij22} + Q_{ij11}) + (Q_{ij21} - Q_{ij11})Z_x + (Q_{ij22} - Q_{ij12})Z_y$$

$$4B_{ij}/h^2 = (1/2)(- Q_{ij22} + Q_{ij11}) + 2(Q_{ij21} - Q_{ij11})Z_x^2 + 2(Q_{ij22} - Q_{ij12})Z_y^2$$

$$12D_{ij}/h^3 = (1/2)(Q_{ij22} + Q_{ij11}) + 4(Q_{ij21} - Q_{ij11})Z_x^3 + 4(Q_{ij22} - Q_{ij12})Z_y^3$$

Since in all of the cases considered in the present investigation, the values of Z_x and Z_y fell within these limits, the above expressions are directly applicable here.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Certain fiber-reinforced materials, especially those with slightly curved
fibers in very soft matrices, exhibit considerably smaller stiffnesses when
loaded in compression than when loaded in tension. Examples are tire cord-
rubber, wire-reinforced solid propellants, and certain soft biological tissues.
For purposes of analysis and design, such materials can be modeled as a bi-
modulus material, i.e., one having one set of stiffnesses when the fiber-
direction strain is tensile and another set when this strain is compressive.
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20. Abstract

Using the fiber-governed bimodulus-material model introduced several years ago by the senior author and verified for cord-rubber composites, the present authors extend their previous work on the deflection of single-layer and cross-ply laminated rectangular plates to circular cylindrical shells of the same construction. A closed-form solution is presented for a thin, freely supported, cylindrically curved panel under sinusoidally distributed loading. Numerical results are presented to show the effect of shell curvature on the neutral-surface positions and deflection.